

### Formula for Zero-inflated Poisson model (ZIP)

(ZIP) represents framework for the analysis the counts data that have distribution with high level of score zero counts than is expected for the Poisson distribution. ZIP assumes that the population involve of two kinds of individual. The first type represents the source of a Poisson-distributed count, which may be zero, whereas the second type always represents the source of a zero count. The distribution has two parameters, the mean of the Poisson distribution ( $\lambda$ ) and the part of individuals that are of the second type ( $p$ ). The mixture distribution formula is (Lambert 1992):

$$P(y_i = k) = \begin{cases} p_i + (1 - p_i)e^{-\lambda_i} & \text{if } k = 0 \\ (1 - p_i) \frac{e^{-\lambda_i} \lambda_i^k}{k!} & \text{if } k = 1, 2, \dots \end{cases}$$

where the parameters  $p_i$ , and  $\lambda$  depend on (vectors of) covariates  $x_i$  and  $z_i$ , respectively through the logit and log links as:

$$\text{logit}(p_i) = x_i \gamma$$

$$\log(\lambda_i) = Z_i \theta$$

**Formula for elastic net (ENET) of ZIP**, defined by the formula (Tang, Xiang, and Zhu 2014):

$$\hat{\beta}_{ENet} = \arg \min \{-L(\beta)\} + \lambda_1 \left\{ \sum_{j=1}^p |\theta_j| + \sum_{j=1}^p |\gamma_j| \right\} + \lambda_2 \left\{ \sum_{j=1}^p \theta_j^2 + \sum_{j=1}^p \gamma_j^2 \right\}$$

### Formula for Bayesian Information Criterion

$$BIC(\lambda) = -2L[\hat{\theta}^{ENET}(\lambda)] + e(\lambda) \log(n)$$